SOLUTION OF EXERCISE # 3.2

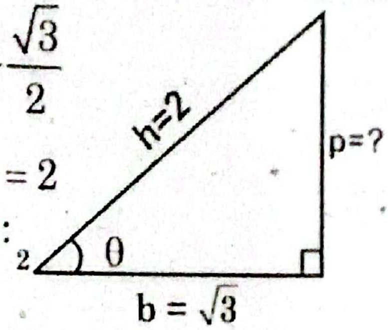
Exercise # 3.2

Q.1: If $\cos\theta = \frac{-\sqrt{3}}{2}$ and the terminal side of the angle lies in the third quadrant, find the remaining trigonometric ratios of θ.

(IIA-2017)

Sol. As $\cos\theta = -\frac{\sqrt{3}}{2}$ then $\boxed{\sec\theta = -\frac{2}{\sqrt{3}}}$

As, $\cos\theta = \frac{\text{base}}{\text{hyp}} = -\frac{\sqrt{3}}{2}$



so, $b = \sqrt{3}$ & $h = 2$

By Pythagoras theorem:

$b^2 + p^2 = h^2$

$(\sqrt{3})^2 + p^2 = (2)^2$

$3 + p^2 = 4$

$p^2 = 4 - 3$

$\sqrt{p^2} = \sqrt{1} \Rightarrow p = 1$

As θ lies in the III-Quad.

As, $\sin\theta = -\frac{p}{h}$

$\Rightarrow \boxed{\sin\theta = -\frac{1}{2}}$

$\Rightarrow \boxed{\operatorname{cosec}\theta = -2}$

As, $\tan\theta = \frac{p}{b}$

$\Rightarrow \boxed{\tan\theta = \frac{1}{\sqrt{3}}}$

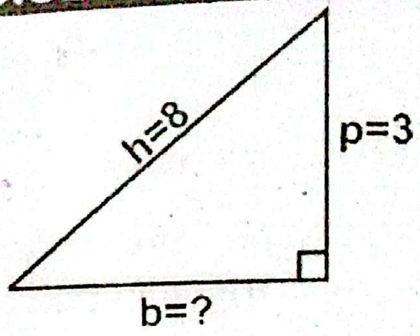
$\Rightarrow \boxed{\cot\theta = \sqrt{3}}$

Q.2: If $\sin\theta = \frac{3}{8}$ and the terminal side of the angle lies in the second quadrant, find the remaining trigonometric ratios of θ.

(IA-2017)

Sol. As $\sin\theta = \frac{3}{8}$ then $\boxed{\operatorname{cosec}\theta = \frac{8}{3}}$

SOLUTION OF EXERCISE # 3.2



As, $\sin\theta = \dfrac{\text{perp}}{\text{hyp}} = \dfrac{3}{8}$

so, $p = 3$ & $h = 8$

By Pythagoras theorem:

$b^2 + p^2 = h^2$

$b^2 + (3)^2 = (8)^2$

$b^2 = 64 - 9$

$b^2 = 55 \quad \Rightarrow \quad b = \sqrt{55}$

As θ lies in the II-Quad.

As, $\cos\theta = -\dfrac{b}{h}$

$\Rightarrow \quad \boxed{\cos\theta = -\dfrac{8}{\sqrt{55}}}$

$\Rightarrow \quad \boxed{\sec\theta = -\dfrac{8}{\sqrt{55}}}$

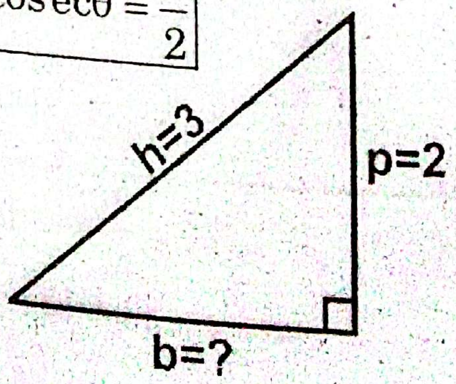
As, $\tan\theta = -\dfrac{p}{b}$

$\Rightarrow \quad \boxed{\tan\theta = -\dfrac{3}{\sqrt{55}}}$

$\Rightarrow \quad \boxed{\cot\theta = -\dfrac{\sqrt{55}}{3}}$

Q.3: **If $\sin\theta = \dfrac{2}{3}$ and the terminal side of the angle lies in the second quadrant, find the remaining trigonometric ratios of θ.**
(IIA-2018), (IIA-2019)

Sol. As $\sin\theta = \dfrac{2}{3}$ then $\boxed{\operatorname{cosec}\theta = \dfrac{3}{2}}$



As, $\sin\theta = \dfrac{\text{perp}}{\text{hyp}} = \dfrac{2}{3}$

so, $p = 2$ & $h = 3$

By Pythagoras theorem:

$b^2 + p^2 = h^2$

$b^2 + (2)^2 = (3)^2$

$b^2 = 9 - 4 \Rightarrow b^2 = 5 \Rightarrow b = \sqrt{5}$

SOLUTION OF EXERCISE # 3.2

As θ lies in the II-Quad.

As, $\cos\theta = -\frac{b}{h}$

$\Rightarrow \boxed{\cos\theta = -\frac{\sqrt{5}}{3}}$

$\Rightarrow \boxed{\sec\theta = -\frac{3}{\sqrt{5}}}$

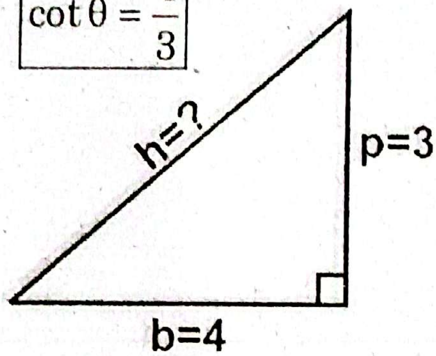
As, $\tan\theta = -\frac{p}{b}$

$\Rightarrow \boxed{\tan\theta = -\frac{2}{\sqrt{5}}}$

$\Rightarrow \boxed{\cot\theta = -\frac{\sqrt{5}}{2}}$

Q.4: If $\tan\theta = \frac{3}{4}$ and the terminal side of the angle lies in the third quadrant, find the remaining trigonometric ratios of θ.

Sol. As $\tan\theta = \frac{3}{4}$ then $\boxed{\cot\theta = \frac{4}{3}}$



As, $\tan\theta = \frac{\text{perp}}{\text{base}} = \frac{3}{4}$

so, $p = 3$ & $b = 4$

By Pythagoras theorem:

$b^2 + p^2 = h^2$

$(4)^2 + (3)^2 = h^2 \Rightarrow 16 + 9 = h^2 \Rightarrow 25 = h^2 \Rightarrow h = 5$

As θ lies in the III-Quad.

As, $\sin\theta = -\frac{p}{h}$

$\Rightarrow \boxed{\sin\theta = -\frac{3}{5}}$

$\Rightarrow \boxed{\operatorname{cosec}\theta = -\frac{5}{3}}$

As, $\cos\theta = -\frac{b}{h}$

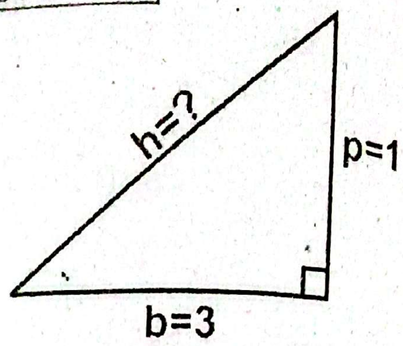
$\Rightarrow \boxed{\cos\theta = -\frac{4}{5}}$

$\Rightarrow \boxed{\sec\theta = -\frac{5}{4}}$

Q.5: If $\tan\theta = -\frac{1}{3}$, and the terminal side of the angle lies in the second quadrant, find the remaining trigonometric ratios of θ.

SOLUTION OF EXERCISE # 3.2

Sol. As $\tan\theta = -\frac{1}{3}$ then $\boxed{\cot\theta = -3}$



As, $\tan\theta = \frac{\text{perp}}{\text{base}} = -\frac{1}{3}$

so, $p = 1$ & $b = 3$

By Pythagoras theorem:

$b^2 + p^2 = h^2$

$(3)^2 + (1)^2 = h^2$

$9 + 1 = h^2$

$10 = h^2 \Rightarrow \sqrt{10} = \sqrt{h^2} \Rightarrow h = \sqrt{10}$

As θ lies in the II-Quad.

As, $\sin\theta = \frac{p}{h}$

$\Rightarrow \boxed{\sin\theta = \frac{1}{\sqrt{10}}}$

$\Rightarrow \boxed{\text{cosec}\theta = \sqrt{10}}$

As, $\cos\theta = -\frac{b}{h}$

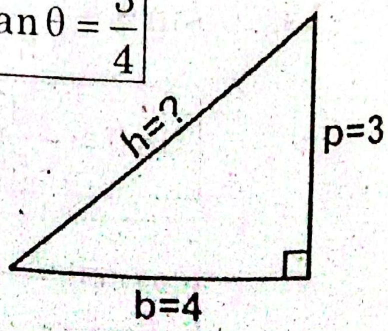
$\Rightarrow \boxed{\cos\theta = -\frac{3}{\sqrt{10}}}$

$\Rightarrow \boxed{\sec\theta = -\frac{\sqrt{10}}{3}}$

Q.6: If $\cot\theta = 4/3$ and the terminal side of the angle is not in the first quadrant, find the remaining trigonometric ratios of θ.

(IA-2021)

Sol. As $\cot\theta = \frac{4}{3}$ then $\boxed{\tan\theta = \frac{3}{4}}$



As, $\tan\theta = \frac{\text{perp}}{\text{base}} = \frac{3}{4}$

so, $p = 3$ & $b = 4$

By Pythagoras theorem:

$b^2 + p^2 = h^2 \Rightarrow (4)^2 + (3)^2 = h^2 \Rightarrow 16 + 9 = h^2$

$25 = h^2 \Rightarrow \sqrt{25} = \sqrt{h^2} \Rightarrow h = 5$

SOLUTION OF EXERCISE # 3.2

As θ lies in the III-Quad.

As, $\sin\theta = -\frac{p}{h}$

$\Rightarrow \boxed{\sin\theta = -\frac{3}{5}}$

$\Rightarrow \boxed{\operatorname{cosec}\theta = -\frac{5}{3}}$

As, $\cos\theta = -\frac{b}{h}$

$\Rightarrow \boxed{\cos\theta = -\frac{4}{5}}$

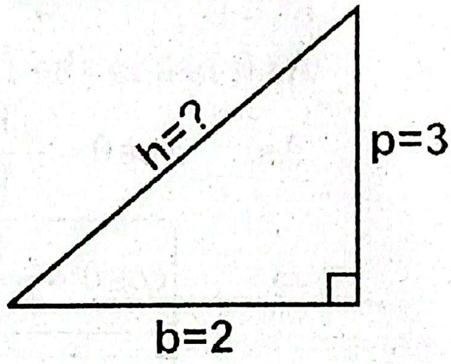
$\Rightarrow \boxed{\sec\theta = -\frac{5}{4}}$

Q.7: If $\cot\theta = \frac{2}{3}$ and the terminal side of the angle does not lies in the first quadrant, find the remaining trigonometric ratios of θ.

Sol. As $\cot\theta = \frac{2}{3}$ then $\boxed{\tan\theta = \frac{3}{2}}$ (IA-2018)

As, $\tan\theta = \frac{\text{perp}}{\text{base}} = \frac{3}{2}$

so, $p = 3$ & $b = 2$



By Pythagoras theorem:

$b^2 + p^2 = h^2$

$(2)^2 + (3)^2 = h^2$

$4 + 9 = h^2$

$13 = h^2 \Rightarrow \sqrt{13} = \sqrt{h^2} \Rightarrow h = \sqrt{13}$

As θ lies in the III-Quad.

As, $\sin\theta = -\frac{p}{h}$

$\Rightarrow \boxed{\sin\theta = -\frac{3}{\sqrt{13}}}$

$\Rightarrow \boxed{\operatorname{cosec}\theta = -\frac{\sqrt{13}}{3}}$

As, $\cos\theta = -\frac{b}{h}$

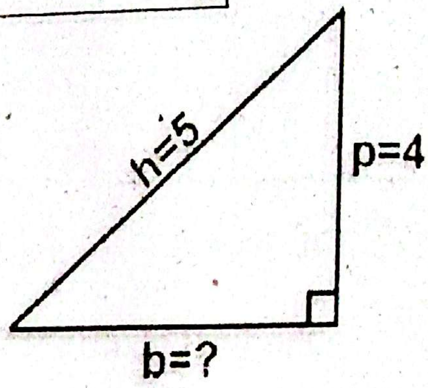
$\Rightarrow \boxed{\cos\theta = -\frac{2}{\sqrt{13}}}$

$\Rightarrow \boxed{\sec\theta = -\frac{\sqrt{13}}{2}}$

SOLUTION OF EXERCISE # 3.2

Q.8: If $\sin\theta = \frac{4}{5}$ and $\frac{\pi}{2} < \theta < \pi$, find the remaining trigonometric ratios of θ.

Sol. As $\sin\theta = \frac{4}{5}$ then $\boxed{\operatorname{cosec}\theta = \frac{5}{4}}$



As, $\sin\theta = \frac{\text{perp}}{\text{hyp}} = \frac{4}{5}$

so, $p = 4$ & $h = 5$

By Pythagoras theorem:

$b^2 + p^2 = h^2$

$b^2 + (4)^2 = (5)^2$

$b^2 = 25 - 16$

$b^2 = 9 \rightarrow b = 3$

As θ lies in the II-Quad.

As, $\cos\theta = -\frac{b}{h}$

$\Rightarrow \boxed{\cos\theta = -\frac{3}{5}}$

$\Rightarrow \boxed{\sec\theta = -\frac{5}{3}}$

As, $\tan\theta = -\frac{p}{b}$

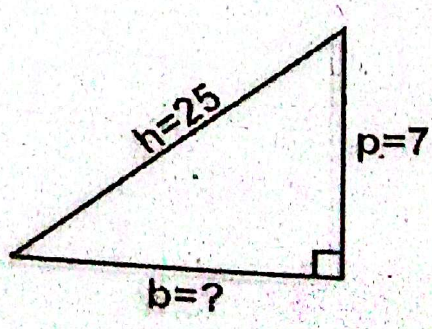
$\Rightarrow \boxed{\tan\theta = -\frac{4}{3}}$

$\Rightarrow \boxed{\cot\theta = -\frac{3}{4}}$

Q.9: If $\sin\theta = 7/25$, find $\cos\theta$, if angle θ is an acute angle.

(IIA-2018), (IIA-2020)

Sol. As, $\sin\theta = \frac{\text{perp}}{\text{hyp}} = \frac{7}{25}$



so, $p = 7$ & $h = 25$

By Pythagoras theorem:

$b^2 + p^2 = h^2$

SOLUTION OF EXERCISE # 3.2

$b^2 + (7)^2 = (25)^2$
$b^2 = 625 - 49$
$b^2 = 576$
$\sqrt{b^2} = \sqrt{576}$
$b = 24$

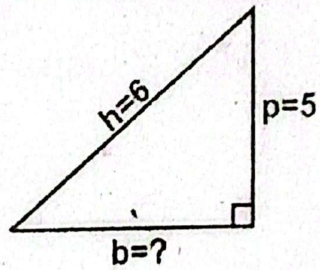
As θ lies in the I-Quad.

As, $\cos\theta = \dfrac{b}{h}$

$\Rightarrow \boxed{\cos\theta = \dfrac{24}{25}}$

Q.10: If $\sin\theta = \dfrac{5}{6}$, find $\cos\theta$, if angle θ is an obtuse angle.

Sol. $\sin\theta = \dfrac{5}{6}$ **(IIA-2021)**



As, $\sin\theta = \dfrac{\text{perp}}{\text{hyp}} = \dfrac{5}{6}$

so, $p = 5$ & $h = 6$

By Pythagoras Theorem

$b^2 + p^2 = h^2$
$b^2 + (5)^2 = (6)^2$
$b^2 = 36 - 25$
$b^2 = 11$
$b = \sqrt{11}$

As θ lies in the II-Quad.

As, $\cos\theta = \dfrac{b}{h}$

$\Rightarrow \boxed{\cos\theta = -\dfrac{\sqrt{11}}{6}}$

Q.11: Prove that:

(i) $\sin\dfrac{\pi}{3}\cos\dfrac{\pi}{6} + \cos\dfrac{\pi}{3}\sin\dfrac{\pi}{6} = \sin\dfrac{\pi}{2}$

Sol. L.H.S. $= \sin\dfrac{\pi}{3}\cos\dfrac{\pi}{6} + \cos\dfrac{\pi}{3}\sin\dfrac{\pi}{6}$

$= \sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$

$= \left(\dfrac{\sqrt{3}}{2}\right)\left(\dfrac{\sqrt{3}}{2}\right) + \left(\dfrac{1}{2}\right)\left(\dfrac{1}{2}\right) = \dfrac{3}{4} + \dfrac{1}{4} = \dfrac{3+1}{4} = \dfrac{4}{4} = 1 \rightarrow (i)$

R.H.S. $= \sin\dfrac{\pi}{2} = \sin 90^\circ = 1 \rightarrow (ii)$

From eq.(i) & eq.(ii) L.H.S. = R.H.S. **Proved**

SOLUTION OF EXERCISE # 3.2

(ii) $4\tan 60^\circ \tan 30^\circ \tan 45^\circ \sin 30^\circ \cos 60^\circ = 1$ **(IIA-2019)**

Sol. $\text{L.H.S.} = 4\tan 60^\circ \tan 30^\circ \tan 45^\circ \sin 30^\circ \cos 60^\circ$

$$= 4\left(\sqrt{3}\right)\left(\frac{1}{\sqrt{3}}\right)(1)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 1 = \text{R.H.S.}$$ **Proved**

(iii) $2\sin 45^\circ + \frac{1}{2}\operatorname{cosec} 45^\circ = \frac{3}{\sqrt{2}}$

Sol. $\text{L.H.S.} = 2\sin 45^\circ + \frac{1}{2}\operatorname{cosec} 45^\circ$

$$= 2\left(\frac{\sqrt{2}}{2}\right) + \frac{1}{2}\left(\frac{2}{\sqrt{2}}\right) = \sqrt{2} + \frac{1}{\sqrt{2}}$$

$$= \frac{(\sqrt{2})^2 + 1}{\sqrt{2}} = \frac{2+1}{\sqrt{2}} = \frac{3}{\sqrt{2}} = \text{R.H.S.}$$ **Proved.**

(iv) $\cos 90^\circ - \cos 30^\circ = -2\sin 60^\circ \sin 30^\circ$

Sol. $\cos 90^\circ - \cos 30^\circ = -2\sin 60^\circ \sin 30^\circ$

$$0 - \frac{\sqrt{3}}{2} = -2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$$

$$-\frac{\sqrt{3}}{2} = -\frac{\sqrt{3}}{2} \Rightarrow \text{L.H.S.} = \text{R.H.S.}$$ **Proved**

(v) $\sin^2\frac{\pi}{6} + \sin^2\frac{\pi}{3} + \tan^2\frac{\pi}{4} = 2$

Sol. $\text{L.H.S.} = \sin^2\frac{\pi}{6} + \sin^2\frac{\pi}{3} + \tan^2\frac{\pi}{4}$

$$= \sin^2 30^\circ + \sin^2 60^\circ + \tan^2 45^\circ$$

$$= \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + (1)^2$$

$$= \frac{1}{4} + \frac{3}{4} + 1 = \frac{1+3+4}{4} = \frac{8}{4} = 2 = \text{R.H.S.}$$ **Proved.**

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Q.12: Prove that: (IIA-2016)

$$\sin^2\frac{\pi}{6}:\sin^2\frac{\pi}{4}:\sin^2\frac{\pi}{3}:\sin^2\frac{\pi}{2}=1:2:3:4$$

Sol. $\text{L.H.S.}=\sin^2\frac{\pi}{6}:\sin^2\frac{\pi}{4}:\sin^2\frac{\pi}{3}:\sin^2\frac{\pi}{2}$

$$=\sin^2 30^\circ:\sin^2 45^\circ:\sin^2 60^\circ:\sin^2 90^\circ$$

$$=\left(\frac{1}{2}\right)^2:\left(\frac{\sqrt{2}}{2}\right)^2:\left(\frac{\sqrt{3}}{2}\right)^2:(1)^2$$

$$=\frac{1}{4}:\frac{2}{4}:\frac{3}{4}:1$$

Multiplying by 4, we have:

$$=4\times\frac{1}{4}:4\times\frac{2}{4}:4\times\frac{3}{4}:4\times 1$$

$$=1:2:3:4=\text{R.H.S.}$$ **Proved**

Q.13: Evaluate:

(i) $\cos 30^\circ\cos 60^\circ-\sin 30^\circ\sin 60^\circ$

(IA-2017), (IIA-2017), (IIA-2018), (IIA-2020)

Sol. $\cos 30^\circ\cos 60^\circ-\sin 30^\circ\sin 60^\circ$

$$=\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)-\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)=\frac{\sqrt{3}}{4}-\frac{\sqrt{3}}{4}=\boxed{0}$$

(ii) $\dfrac{\tan 60^\circ-\tan 30^\circ}{1+\tan 60^\circ\tan 30^\circ}$

Sol. $$\frac{\tan 60^\circ-\tan 30^\circ}{1+\tan 60^\circ\tan 30^\circ}=\frac{\sqrt{3}-\frac{1}{\sqrt{3}}}{1+\left(\sqrt{3}\right)\left(\frac{1}{\sqrt{3}}\right)}=\frac{\frac{\left(\sqrt{3}\right)^2-1}{\sqrt{3}}}{1+1}$$

$$=\frac{3-1}{2\sqrt{3}}=\frac{2}{2\sqrt{3}}=\boxed{\frac{1}{\sqrt{3}}}$$